Absolute Values / Inequalities

On the calculator: ABS( )

Notation $|x|$

If $|x| = 4$ then $x = 4$ or $x = -4$

When would this be useful?

Solving Equations

eg 1) $|2x| = 10$

$\Rightarrow \quad 2x = 10 \quad \text{or} \quad 2x = -10$

$\Rightarrow \quad x = 5 \quad \text{or} \quad -5$

eg 2) $|x+1| = 7$

$\Rightarrow \quad x+1 = 7 \quad \text{or} \quad x+1 = -7$

$\Rightarrow \quad x = 8 \quad \text{or} \quad 6$

eg 3 $|3-x| = x-1$

$\Rightarrow$ either $3-x = x-1$ or $3-x = -(x-1)$

$\Rightarrow$ $3-x = x-1$

$2x = 4$

$x = 2$

and $3-x = -x+1$

$\Rightarrow$ $4 = 0x$

x is undefined

So no solution

Page 29 Ex 2.2.1

Qs 5 onwards – you choose... But not question 7c onwards.

Absolute Values + Inequalities

a) $|x| < a$

$\Rightarrow \quad -a < x < a$

b) $|x| > a$

$\Rightarrow \quad x < -a \quad \text{or} \quad x > a$

eg. $3|6-4x|+1 > 10$

$|6-4x| > 3$

$\Rightarrow \quad 6-4x > 3$ ① or $6-4x < -3$ ②

So, ① $6-3 > 4x$

$x < \frac{3}{4}$

and ② $4x > 9$

$x > \frac{9}{4}$

eg2 Find $\{x : |x| > 1-2x\}$

we have $x > 1-2x$ ... (1) or $x < -(1-2x)$ ... (2)

(1) $3x > 1$

$x > \frac{1}{3}$

(2) $x < -1+2x$

$x > 1$

Is this correct?

Sure?

When doing an inequation involving ABS we should consider what values x can take at the start:

eg $|x+2| > 4 - 8x$

when $x > -2$ then $x + 2 > 4 - 8x$

$9x > 2$

$x > \frac{2}{9}$

Since x has to be greater than -2, this is correct.

when $x < -2$

$x + 2 < -(4 - 8x)$

$x + 2 < -4 + 8x$

$6 < 7x$

$x > \frac{6}{7}$

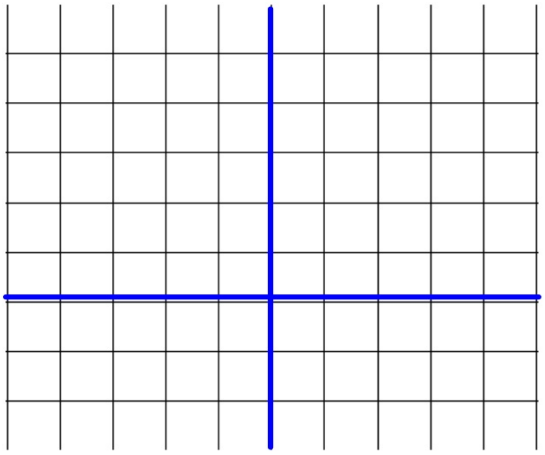
Since x has to be less than -2 this is not a viable solution

Page 32 Q 5 onwards.

7c

$|2x + a| = |x| - a$ where $a > 0$

Sketch:



Eg 8a

$$|x+1| + |x-1| = 3$$

So we have:

$x+1$ if $x \geq -1$ and $x-1$ if $x \geq 1$

$-(x+1)$ if $x < -1$ $\quad$ $-(x-1)$ if $x < 1$

So, if: $x \geq 1 \Rightarrow x+1+x-1 = 2x$

$-1 \leq x < 1 \Rightarrow x+1-(x-1) = 2$

$x < -1 \Rightarrow -(x+1)-(x-1) = -2x$

$\Rightarrow |x+1| + |x-1| = 3$

$\Rightarrow 2x = 3$ or $-2x = 3$

$\Rightarrow x = \frac{3}{2}$ or $-\frac{3}{2}$

8c $\left|\frac{x}{a}\right| \geq x + a \; ; \; 0 < a < 1$

If $x \geq 0$ then $\frac{x}{a} \geq x + a$

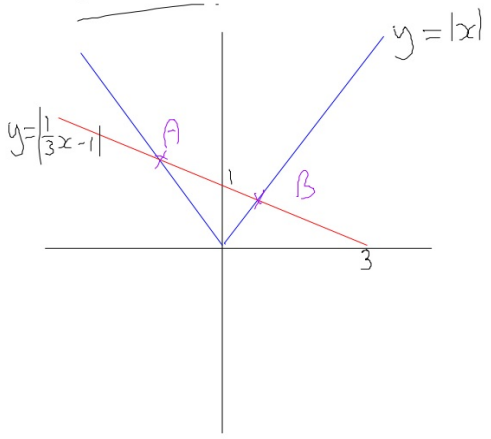
$$x \geq ax + a^2$$
$$x - ax \geq a^2$$
$$x(1-a) \geq a^2$$
$$x \geq \frac{a^2}{1-a}$$

and, if $x < 0$

$$\frac{x}{a} \leq -x - a$$
$$x \leq -ax - a^2$$
$$x + ax \leq -a^2$$
$$x(1+a) \leq -a^2$$
$$x \leq \frac{-a^2}{1+a}$$

9b) $\left|\frac{1}{3}x - 1\right| > |x|$

Sketch.



so we want between the 2 points A and B.

for $x < 0$

$-\left(\frac{1}{3}x - 1\right) = -x$

$x = -\frac{3}{2}$ ..... pt. A

for $x > 0$

$-\left(\frac{1}{3}x - 1\right) = x$

$\frac{1}{3}x + 1 = x$

$x = \frac{3}{4}$ .... pt B

$-\frac{3}{2} \leq x \leq \frac{3}{4}$